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THE USE OF CORRELATION DIMENSION IN CONDITION MONITORING OF SYSTEMS WITH CLEARANCE

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In this paper the application of embedding space and fractal dimension estimation to monitor the condition of a system with clearance is considered by examining the effect on the correlation dimension of variations of the gap clearance in a bearing system. The model analysed represents an elastically supported rotor and stator, subject to excitation by an imbalance, which is restricted to two-dimensional linear movement perpendicular to the axis of rotation. From full-phase space calculations it is found that as the clearance between the rotor and stator is increased, there is a discernible decrease in the value of the correlation dimension. Thus, it may be feasible to determine changing events such as gap clearance by monitoring the correlation dimension of the system. Employing the conventional method of delays results in substandard estimates for the correlation dimension. However, modifying the method of delays by incorporating values from an additional observable into the vectors used for reconstructing the phase space produce results in accordance with those from the full-phase space.

1. INTRODUCTION

Condition monitoring, or machine health monitoring, is the extraction of machine vibration data, machine performance data and/or other relevant information from plant or machinery whilst in operation, with the objective of identifying the initial indications of component failure. Once detected, pre-failure damage can then be located and treated accordingly to avoid an all-out breakdown of the machinery, making considerable savings possible both financially and in machine downtime. Traditionally, the method used for such monitoring processes involve analyses in the time and frequency domains, covering signal processing and Fourier analysis techniques respectively. More recently, more sophisticated tools such as neural networks and wavelet techniques have been employed. In general, plant or machinery will have non-linear components which contribute to the dynamics of the entire system and which may, under certain conditions, produce chaotic vibrations. In such circumstances, chaos techniques may present alternative methods for the condition monitoring of such equipment. The work reported in this paper explores this idea through the investigation of the use of the correlation dimension for the condition monitoring of systems with clearance.

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It has already been ascertained that dissipative dynamical systems which exhibit chaotic behaviour often have an attractor in phase space which is termed as "strange". These so-called "strange attractors" have been found to be non-periodic and unpredictable over long time scales due to the system's sensitive dependence on its initial conditions. For this reason chaotic dynamics have been found to be most easily understood when viewed from a phase-space perspective [1–3]. Systems with clearance can, under specific circumstances, experience chaotic motion and therefore may also have a "strange attractor" in phase space. As most experimental situations or condition monitoring programmes will yield data for only some of the observables which contribute to the dynamics of the system, the actual phase space of the system cannot be obtained. However, by using the method of delays, the system's phase-space attractor can be reconstructed from the knowledge of as little as one of the observable of the system [4]. In this pseudo-phase space or indeed the actual-phase space, analytical techniques such as Lyapunov exponents and fractal dimensions can be applied in an effort to predict the system's behaviour with time.

The calculation of fractal dimensions from the pseudo-phase space (also known as embedding space) has been comprehensively studied by Grassberger and Procaccia [5]. Sauer and Yorke [6] have furthered this study by researching the minimum number of delay co-ordinates required to give a faithful representation of the full-phase-space attractor, while the practical problems associated with the dimension computations have also considered in two papers by Ding *et al.* [7, 8]. In a more recent paper by Jedynak *et al.* [9], the failure of dimensionality calculations in the pseudo-phase space due to the application of the method beyond its intrinsic limitations is discussed.

The following paper considers the application of embedding space and correlation dimension estimation to monitor the condition of a system with clearance, as the gap clearance is varied. It will be shown that because the coupling to the non-linearity is more dominant in the x direction, inexact reconstructions of the phase spaces can arise when using the y direction motion for the reconstructions in this type of system. This becomes more apparent in the results obtained for the correlation dimension as the gap clearance is increased, due to the weakening of the coupling between the two motions. Problems with phase-space reconstructions for discontinuous systems have previously been documented in a paper by Feeny and Liang [10]. Reconstruction failure in that case was attributed to non-smooth processes associated with systems exhibiting stick–slip dynamics. Successful reconstructions were achieved by the addition of an extra observable in the delay vectors when reconstructing the phase spaces. In this paper similar technique are presented to show the improvement in the quality of the embedding.

2. REVIEW OF DYNAMIC MODEL

2.1. MODEL DESCRIPTION

The model used for the subsequent analysis is shown in Figure 1. It is similar to that which has been analyzed both numerically and experimentally prior to the work researched here but, in addition, includes the effect of friction between the

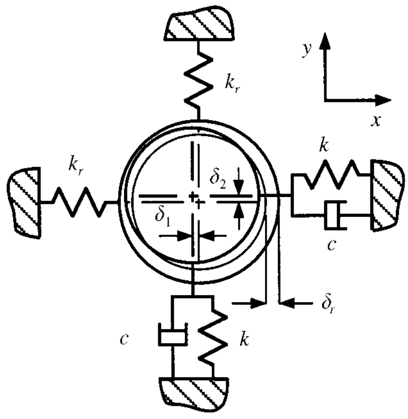

Figure 1. Dynamic model of the rig.

rotor and stator during sliding contact [11]. The model represents an elastically supported rotor, subject to excitation by imbalance, which is restricted to two-dimensional motion in a plane perpendicular to the axis of rotation. The mass eccentricity of the rotor is defined as ρ, and the mass is free to vibrate inside a gap clearance δ_r. Once the rotor has traversed this gap clearance, which could be representative of a clearing effect such as a bearing or a malfunctioning squeeze film bearing, contract is made with the elastically supported ring, which is modelled as having zero mass. The stiffness ratio of the ring support relative to the rotor support is approximately 30:1, and δ_1 and δ_2 allow the rotor to be offset within the clearance to model gravity or other effects. The inherent damping in the rotor system and the additional damping arising from contact between the rotor housing and the ring support are represented by c_1 and c_2 respectively.

2.2. EQUATIONS OF MOTION

The equations of motion for the above system are presented below. For the case where there is no contact between the rotor and housing, the motion is governed by

$$\ddot{x} + 2\nu_1\omega_1\dot{x} + \omega_1^2 x = \rho\Omega^2 \cos \Omega t, \tag{1}$$

$$\ddot{y} + 2\nu_1\omega_1\dot{y} + \omega_1^2 y = \rho\Omega^2 \sin \Omega t. \tag{2}$$

When there is contact between the rotor housing and ring, these equations become

$$\begin{aligned}\ddot{x} + 2\nu_1\omega_1\dot{x} + 2\nu_2\omega_2\left\{\dot{x} - \delta_r(y-\delta_2)\left(\frac{V_{Tang}}{R^2}\right)\right\} \\ + \omega_1^2 x + \omega_2^2\left(1-\frac{\delta_r}{R}\right)(x-\delta_1) \\ - \operatorname{sgn}(V_{Tang})\mu\omega_2^2\left(1-\frac{\delta_r}{R}\right)(y-\delta_2) \\ = \rho\Omega^2 \cos t,\end{aligned} \tag{3}$$

$$\ddot{y} + 2v_1\omega_1\dot{y} + 2v_2\omega_2\left\{\dot{y} - \delta_r(x-\delta_1)\left(\frac{V_{Tang}}{R^2}\right)\right\}$$
$$+ \omega_1^2 x + \omega_2^2\left(1 - \frac{\delta_r}{R}\right)(y-\delta_2) \tag{4}$$
$$+ \operatorname{sgn}(V_{Tang})\mu\omega_2^2\left(1-\frac{\delta_r}{R}\right)(x-\delta_1)$$
$$= \rho\Omega^2 \sin \Omega t,$$

where

$$\omega_1 = \sqrt{\frac{k_1}{m}}, \quad \omega_2 = \sqrt{\frac{k_2}{m}}, \quad v_1 = \frac{c_1}{2m\omega_1}, \quad v_2 = \frac{c_2}{2m\omega_2}$$

$$R = \sqrt{(x-\delta_1)^2 + (y-\delta_2)^2} \quad \text{and} \quad V_{Tang} = \frac{\dot{y}(x-\delta_1)}{R} - \frac{\dot{x}(y-\delta_2)}{R}.$$

All model simulations were run at shaft speeds of either 1260 or 2340 r.p.m. with the following parameter values; $m = 8{\cdot}9$ kg, $k_1 = 79$ kN/m, $k_2 = 2345$ kN/m, $v_1 = 0{\cdot}089$, $v_2 = 0{\cdot}001$, $\rho = 60{\cdot}6 \times 10^{-6}$ m, and $\mu = 0{\cdot}04$, with $\delta_1 = \delta_r = 0{\cdot}003$, 0·005 or 0·00075 m for the three different gap clearances to be analyzed and $\delta_2 = 0$.

2.3. NUMERICAL INTEGRATION

The numerical techniques used to integrate the equations of motion are based on the classical forth order Runge–Kutta algorithm. As the equations of motion change abruptly when the clearance is traversed and contact is made with the secondary spring, care is required in the application of numerical integration techniques if a converged solution is to be obtained. Arbitrary application of a fixed time step fourth order Runge–Kutta algorithm to non-smooth and discontinous systems has been shown to result in solutions which lose convergence after the first discontinuity [12]. To obviate this problem, the algorithms used to produce the data for this study incorporate routines to track the displacements and to locate accurately in time when the contact with the secondary spring was made or broken. This ensures that the convergence of the solution is retained. A hierarchy of three interpolation subroutines from quadratic to linear and finally to bisection was used to detect the points of transition. In well-conditioned cases, a number of applications of the quadratic routine was used to give the contact point to a known spatial tolerance. In badly conditioned cases, linear interpolation was used and in the case of “grazing” type contact recourse was made to the bisection algorithm. Further details of the algorithm may be found in references [13, 14].

3. FRACTAL DIMENSION

3.1. CORRELATION DIMENSION ESTIMATION

Strange attractors are typically characterised by fractal dimensionality, of which the correlation dimension is the most readily computed with rapid convergence. It has also been found to give a more relevant measure of the strangeness, as it is sensitive to the dynamical process of coverage of the phase space since it takes into account the frequency with which trajectories visit different regions of the attractor [15]. The correlation dimension [4] is obtained by firstly considering correlations between points taken from a long time series on the attractor to compute what is known as the correlation function. A sphere (or cube) of radius (or length) r is placed at each point x_i on the attractor and the number of points falling within each sphere is summed, that is,

$$C(r) = \lim_{N \to \infty} \frac{1}{N} \sum_{t, i \neq j}^{N} \sum_{j}^{N} H(r - |x_i - x_j|), \tag{5}$$

where $H(s)$ is the Heaviside function. $H(s) = 1$ if $s > 0$ and $H(s) = 0$ if $s < 0$; s being the distance between pairs of points (given by $r - |x_i - x_j|$).

The function $C(r)$ has been found to behave as a power law [4], which is dependent on r as $r \to 0$; that is,

$$C(r) = 0(r^v). \tag{6}$$

The correlation dimension, commonly denoted by D_2 is then v and can be defined by using the slope of the $\log C$ versus $\log r$ curve:

$$D_2 = \lim_{r \to 0} \frac{\log C(r)}{\log r}. \tag{7}$$

3.2. ACTUAL CORRELATION DIMENSION

The rotor system illustrated in Figure 1 has been extensively studied both experimentally and numerically [12]. As all of the observables which contribute to the dynamics of the system are known from previous analyses, the correlation dimension can be calculated directly from the full-phase-space attractor. The contributing observables are the x and y direction motions and velocities together with the phase angle from the excitation force. To investigate the effect that changing the magnitude of clearance has on the correlation dimension, the system was analysed running at two distinct shaft speeds and was considered for the cases of the clearance being 0·3, 0·5, and 0·75 mm. Table 1 presents the results for the correlation dimensions computed for each of these three different cases for the system running at the two chosen shaft speeds of 1260 and 2340 r.p.m. Examination of these results clearly shows that for both the shaft speeds analysed, as the clearance is increased, there is a noticeable drop in the magnitude of the correlation dimension. Hence, it should be viable to determine changes in the clearance through monitoring the correlation dimension associated with the system.

TABLE 1

Correlation dimension values computed from the full-phase space, for the rotor system illustrated in Figure 1

Gap clearance (mm)	Points in the phase space	Estimated correlation dimension	
		1260 r.p.m.	2340 r.p.m.
0·3	20 000	4·1968	2·7356
0·5	20 000	3·8119	2·5286
0·75	20 000	3·1800	2·2309

3.3. RESULTS OF EMBEDDING FOR 2-d.o.f. ROTOR

As was noted in section 3.1, for the rotor system investigated, all observables contributing to the dynamics of the system were known from previous analyses. In experimental set-ups, or monitoring programmes, however, this in-depth knowledge will not be known, and generally, data from only one state variable may be available or readily recorded. To compensate for this, as little as one observable can be used to reconstruct the phase space by applying a technique known as the method of delays [4]. Reconsidering the same three cases that were used to construct the full-phase space, attempts were made to see if a reliable estimate of the correlation dimension could be determined from the pseudo-phase space. The system was again analysed at the two shaft speeds of 1260 and 2340 r.p.m. Data sets of 20 000 points were used for the analysis, with the length of time delay used in the reconstruction vectors being equivalent to one-quarter of the forcing period of the system. This was determined by locating the first zero crossing when the autocorrelation function is applied to the system [9]. Both the x and y direction motions were considered separately for reconstructing the phase-space attractors, and the correlation dimension was calculated for embedding dimensions ranging from 3 to 12, the embedding dimension being the dimension of the reconstructed phase space. For example, an embedding dimension of 5 will reconstruct the attractor in a five dimensional pseudo-phase space. It was not thought worthwhile to consider an embedding space of smaller dimension as the rotor system being analysed requires two degrees of freedom, i.e., a five-dimensional phase space to describe it. Figures 2 and 3 illustrate the behaviour of the correlation dimension in the x and y directions, respectively, with increasing pseudo-phase-space dimension for the lower of the two shaft speeds analysed, i.e., 1260 r.p.m. Examining Figures 2 and 3, it can be seen that a saturated value is obtained for the correlation dimension from both the x and y direction motions in all the three cases analysed. The approximate values obtained for the saturated correlation dimensions at both the shaft speeds analysed are summarized in Table 2. On closer inspection, however, and comparing these values to the values obtained for the correlation dimension from the full phase spaces (Table 1), it can be seen that although a very reasonable estimate for the correlation dimension is obtained in all three cases at

TABLE 2

Saturated values for the correlation dimension computed from the pseudo-phase space using the standard method of delays for the rotor system illustrated in Figure 1

Gap clearance: x direction (mm)	Estimated correlation dimension		Gap clearance: y direction (mm)	Estimated correlation dimension	
	1260 r.p.m.	2340 r.p.m.		1260 r.p.m.	2340 r.p.m.
0·3	4·4042	2·7102	0·3	4·4358	2·7187
0·5	3·7812	2·5240	0·5	3·4024	2·3191
0·75	3·3751	2·2138	0·75	2·8894	1·9279

both shaft speeds for the x direction motion, this is true only in the 0·3 mm case for the y direction motion. For the y direction motion in the 0·5 and the 0·75 mm cases, the saturated values obtained for the correlation dimension are considerably smaller than those obtained form the x direction motion with the difference appearing to increase progressively as the clearance is increased.

It is clear from this that some problem occurs during the phase-space reconstructions from the y direction motions in the 0·5 and the 0·75 mm cases, producing poor-quality reconstructed "attractors" and consequently, failure of the dimensionality calculations. It is thought that this may be the consequence of variations in the strength of the coupling between the x and y direction motions. As mentioned previously, the coupling between the x and y direction motions and the non-linearity is more dominant in the x direction motion. This is a consequence of $\delta_1 = \delta_r$ and $\delta_2 = 0$. As the clearance is increased, the coupling becomes progressively weaker in the y direction. Furthermore, with the coupling becoming prominently weaker in the y direction, less information is available about the x direction motion from the y direction motion, resulting in substandard reconstructions. The x direction reconstructions, however, are not notably affected by the increase in the clearance, due to the coupling having greater influence through this motion. Therefore, even with weaker coupling between the motions and the non-linearity, sufficient information about the y direction motion still exists to provide reliable reconstructions.

In the literature concerning the calculation of the correlation dimension from an embedding space, a number of different algorithms exist to aid in deciding how many dimensions are required for constructing the pseudo-phase space to give a faithful representation of the actual phase-space attractor. It has been shown [16], that for a one-to-one correspondence from the original phase space to the embedding space, $m = 2N + 1$ dimensions are required, where m is the dimensionality of the pseudo-phase space and N is the dimension of the space in which the original attractor lies. However, it has also been conjectured [6] that for the case of the correlation dimension, the dimension is preserved for $m > D_2$ dimensions, where D_2 is the correlation dimension, even though the one-to-one property may not be satisfied. Re-examining Figures 2 and 3, it is clear that

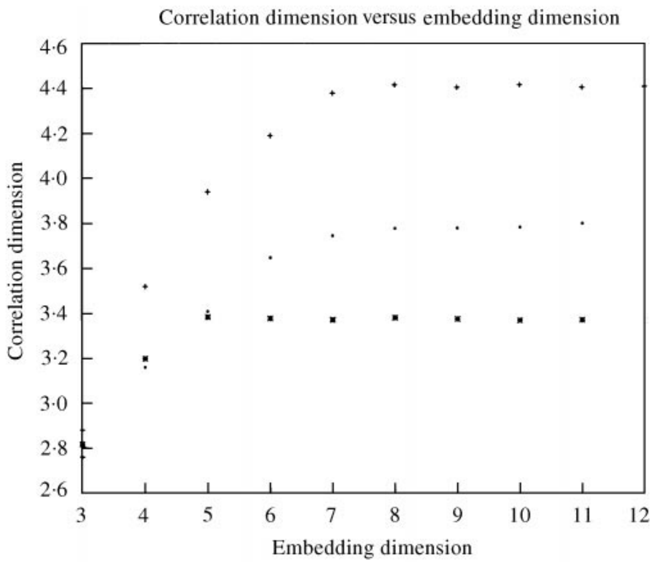

Figure 2. Correlation dimension versus embedding dimension: x direction motion: +, 0·3 mm – x; ·, 0·5 mm – x; ∗, 0·75 mm – x.

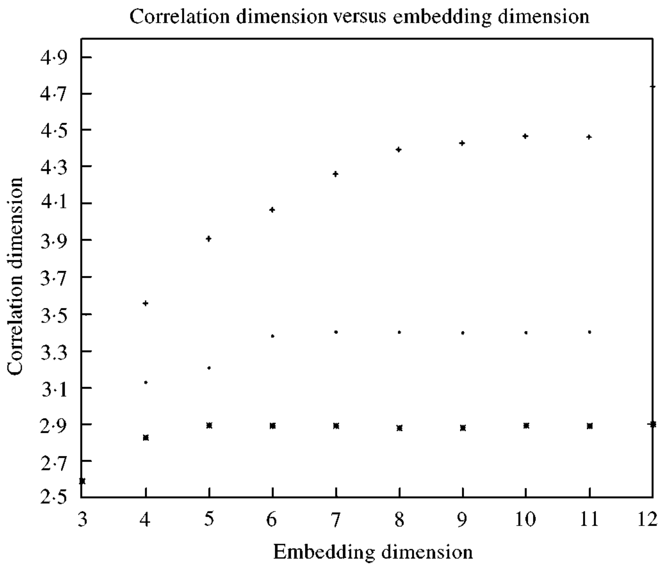

Figure 3. Correlation dimension versus embedding dimension: y direction motion: +, 0·3 mm – y; ·, 0·5 mm – y; ∗, 0·75 mm – y.

a saturated value is not obtained until an embedding space of at least 6 or more dimensions is reached. This may, however, be improved by using much greater data sets for the calculation of the correlation dimension, to the detriment of computation time.

3.4. FURTHER ANALYSIS

As was mentioned earlier, work carried out on systems containing stick–slip dynamics [10] experienced problems when reconstructing the phase-space attractors using the method of delays. In order to make the reconstructions 'true" embeddings, an additional observable (namely, the phase of the excitation in that case) was incorporated into the delays for the reconstructions. If the full-phase space is considered to be the extreme at one end of a spectrum with the pseudo-phase space being the extreme at the opposite end of the spectrum, it can be reasoned that varying combinations of embedding spaces using more than just purely one observable pertaining to the system will generate a form of pseudo-phase space somewhere between the two extremes. Furthermore, if the pure form of pseudo-phase space is in some way not completely representative of the full-phase space of the system, then some combinations between the two extremes may prove to produce more adequate representative embedding spaces. Hence, on incorporating a supplementary observable into the delay measurements, enough information about the other motion may now be available for constructing good-quality attractors, as this would bring the reconstructions marginally closer to that of the full-phase-space attractors.

In order to modify the embeddings described in section 3.3 to determine the effect this will have on the calculations of the correlation dimension in the three cases considered above, incorporation of a y value was included in the x direction reconstruction and *vice versa* for the y direction motions. For example, considering a four dimensional embedding in the x-direction, for the general case the m-dimensional embedding vector would be as follows:

$$x_m = \{x(t), x(t + m\tau), x(t + (m + 1)\tau), x(t + (m + 2)\tau)\}, \tag{8}$$

where $m = 1, 2, 3\ldots$. To incorporate an additional observable equation (8) would become

$$x_m = \{x(t), x(t + m\tau), x(t + (m + 1)\tau), y(t)\}. \tag{9}$$

When this technique was applied to the system considered in section 3.3, very little change, if any at all, occurred in the values obtained for the x direction motion compared to those presented in Table 2. However, in the y direction, significant increases were experienced in the dimensional values from the y direction motion. Unfortunately, these new results were still discernibly lower than the values obtained from the x direction motion. The next obvious step was to incorporate two values from the additional observable into the delay co-ordinates. Applying this, equation (9) for a four-dimensional space then becomes

$$x_m = \{x(t), x(t + m\tau), y(t), y(t + m\tau)\}. \tag{10}$$

Re-analysing the same three cases previously considered, a very desirable outcome was achieved. Not only were the values obtained for the correlation dimensions from the x direction motions of greater accuracy when compared to the values from the full-phase space, but the y direction results were now also of the

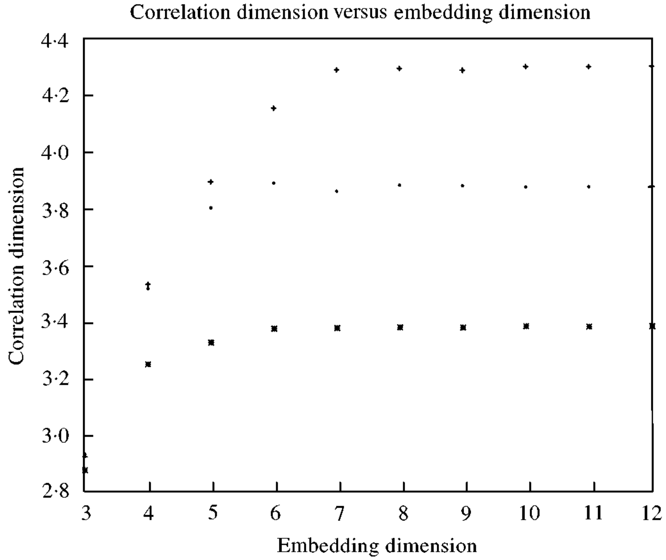

Figure 4. Correlation dimension versus embedding dimension: predominantly x direction: +, 0·3 mm — xy; ·, 0·5 mm — xy; ∗, 0·75 mm — xy.

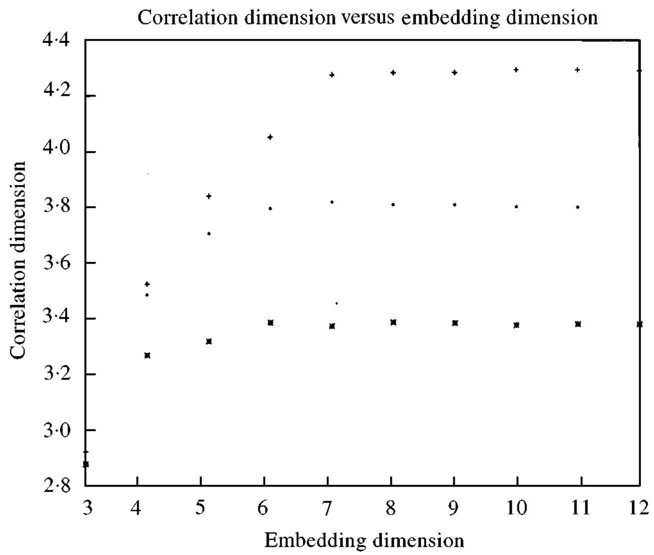

Figure 5. Correlation dimension versus embedding dimension: predominantly y direction: +, 0·3 mm — yx; ·, 0·5 mm — yx; ∗, 0·75 mm — yx.

same magnitude as the x direction motions and similar to the full-phase space results. Examples of the results obtained at the lower shaft speed of 1260 r.p.m. acquired from the x and y direction reconstructions are presented in Figures 4 and 5 respectively when two values from an additional observable are introduced into the delay vectors. As can be seen from Figure 4 and 5, the introduction of two

TABLE 3

Saturated values for the correlation dimension computed from the pseudo-phase space using the modified method of delays for the rotor system illustrated in Figure 1

Gap clearance: Predominantly x direction (mm)	Estimated correlation dimension		Gap clearance: Predominantly y direction (mm)	Estimated correlation dimension	
	1260 r.p.m.	2340 r.p.m.		1260 r.p.m.	2340 r.p.m.
0·3	4·2939	2·7693	0·3	4·2882	2·7660
0·5	3·8756	2·5646	0·5	3·8064	2·5631
0·75	3·3843	2·2427	0·75	3·3800	2·2415

values from an additional observable into the delay vectors for the reconstructions has marginally improved the quality of the reconstructed "attractors" from the x direction motion, with the improvement being much more significant for the y direction reconstructions. Therefore, applying this technique results in better estimates of the correlation dimension compared to those of the full-phase spaces (Table 1) from both the predominantly x and y direction reconstructions. This can be seen more clearly by referring to the approximate saturated values for the correlation dimension presented in Table 3.

It should be noted that this modified technique does not discernibly effect the embedding dimensions at which saturation of the correlation dimension occurs. As was mentioned earlier, it is thought that the process of dimension estimation may be significantly improved by using considerably larger data sets. Consequently, this may also enhance saturation of the correlation dimension within the scope of any of the algorithms conjectured for calculating the minimum embedding dimensions required for faithful reconstructions.

3.5. FALSE NEAREST-NEIGHBOUR TESTS

To assess the legitimacy of the "attractors" constructed for the rotor system of Figure 1, false nearest-neighbour tests [17], were applied to all of the reconstructed attractors analysed in sections 3.3 and 3.4. False nearest-neighbour tests are primarily used to determine the embedding dimension required to obtain a "true" embedding. "True" in this sense means that all pseudo-phase-space points in the reconstructed attractor are genuine neighbours to each other. If an "attractor" is reconstructed using too small an embedding dimension, the "attractor" will cross over itself, and will only unfold with increasing embedding dimension. Thus, if a pseudo-phase space point's nearest neighbour in the smaller embedding dimension becomes remote in the larger embedding dimension it will be regarded as being a false neighbour. The appropriate embedding dimension is achieved when the dimension where no false nearest neighbours are present is reached.

Figure 6 shows the typical characteristic obtained on applying the false nearest-neighbour tests to both the straight embeddings of the rotor system and

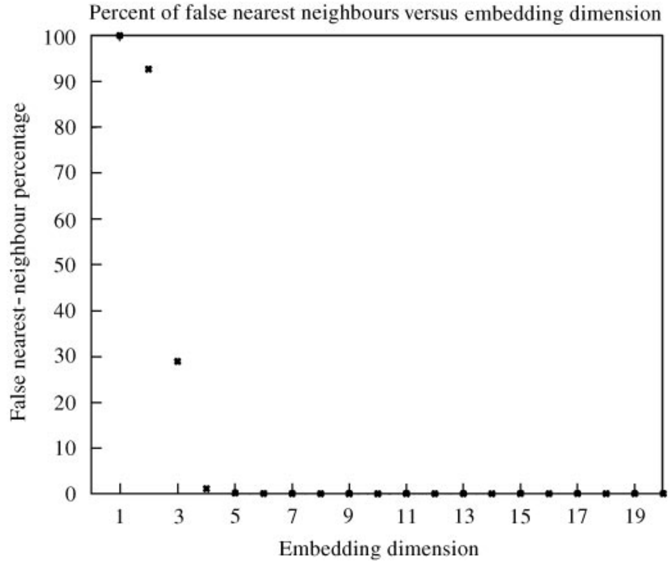

Figure 6. False nearest-neighbour percentage versus embedding dimension for the 0·75 mm *y* direction embedding.

also the modified embeddings. All of the tests showed that the reconstructions were "true" in the sense that by roughly seven dimensions, the "attractors" were fully unfolded, containing only points that were genuine neighbours to each other. However, in the case of the "attractors" reconstructed from the 0·5 and the 0·75 mm *y* direction gap clearances, it is suggested from the failure of the correlation dimension calculations that for these cases the "attractors" were not "true" in the sense that they did not represent the system's proper motion. A "true" attractor, in both the false nearest-neighbour sense and representative of the system's proper motion, was only obtained when the "attractors" were reconstructed using the modified embedding technique. Although, as this work has shown, false nearest-neighbour test can be successfully applied to determine the number of embedding dimensions required to give "true" embeddings in the sense that the attractor is fully unfolded, it has been highlighted that additional analysis tools should also be applied in order to be confident that the "attractor" is representative of the true attractor, constructed from the full-phase space.

3.6. RESULTS FOR S-d.o.f. SYSTEM

In order to discern how the application of embedding space to compute the correlation dimension for a simple system compares to that of the more complex system of Figure 1 a single-degree-of-freedom system was also analysed in the same way as the rotor system of Figure 1. For this analysis the system was run at the lower of the two shaft speeds for which the two-degree-freedom rotor system was considered.

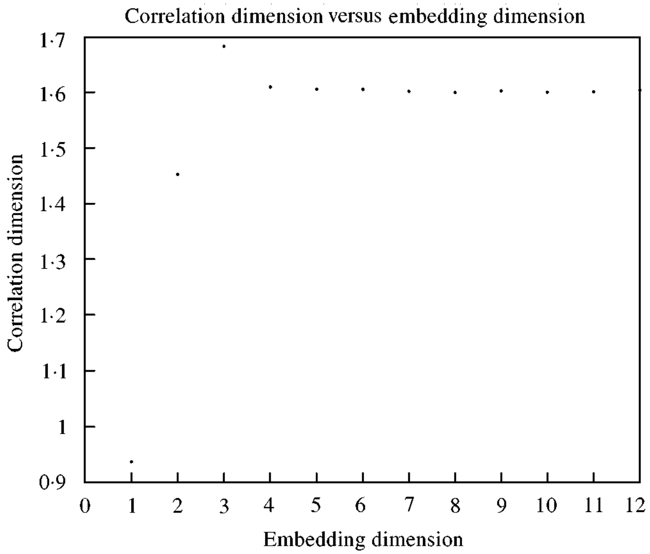

Figure 7. Correlation dimension versus embedding dimension S-d.o.f. system.

The correlation dimension was, as before, computed first from the full-phase space, and then from the pseudo-phase space. The value obtained for the correlation dimension from the phase space was approximately 1·6044. The results obtained from the pseudo-phase space, for a range of embedding dimensions from 1 through to 9, are presented in Figure 7. On examining Figure 7, it can be seen that a saturated value for the correlation dimension (1·605) occurs by as little as 5 embedding dimensions with this value being a very good estimate to that obtained from the full-phase space. Hence, arbitrary application of embedding space to a system of this type proceeds with no problems, unlike the previous systems. This tends to confirm that the primary problems in the case of the rotor system is not one of non-smoothness, but rather the effect of coupling between the motion and the non-linearity.

3.7. DISCUSSION

On comparing the applicability of embedding space to both the single-degree-of-freedom system and the rotor system, it can be seen that for the single-degree-of-freedom system, arbitrary application of this method would be acceptable as no problems arise with the reconstructions. Any calculation carried out in this pseudo-phase space would, therefore, give reliable results. For the rotor system, however, problems can occur with the reconstructions (due to the decrease in strength of the coupling between the non-linearity and the x and y direction motions as the gap clearance is increased) making them topologically dissimilar to those of the real phase space. Therefore, direct application of embedding space in this case may result in incorrect calculation of the correlation dimension from the pseudo-phase space. In the case of this study, in order to modify the reconstructions so that they are topologically similar, an additional observable has to be incorporated into the delay measurement vectors used to create the embedding space.

Unfortunately, in practice, the applicability of embedding space to a system will not be known *a priori* and therefore sufficient care and caution must be taken when reconstructing a system's attractor in order to calculate any properties of the system. As mentioned in section 3.5, the use of false nearest-neighbour tests will give some guidance as to whether problems may arise with the reconstructions and hence a better knowledge as to the applicability of embedding space may be determined from such tests.

In this study, the model assumes a stiffness ratio of approximately 30:1 between the rotor support relative to the ring support. For this stiffness ratio a variety of shaft speeds was available at which the system could exhibit chaotic motion. If, for this system, this ratio was to be decreased, it would become progressively more difficult for the system to exhibit this kind of motion and instead the system would tend to display either periodic or quasi-period motion, irrespective of the clearance or the shaft speed. The resulting value of the correlation dimension would always be either 1 or 2 respectively. Hence, this stiffness ratio was chosen to produce the desired effect of the system exhibiting chaotic motion, so that as the clearance was altered the system could be analysed to determine if a trend existed between changes in clearance and the correlation dimension of the system.

The system considered in this paper represents a non-linear rotor stator system with somewhat exaggerated magnitudes of clearances. From scaling considerations, it has been shown that dynamically similar responses can be elicited at similar values of ρ/δ_r if all other system parameters are held fixed [18]. It should therefore be possible to detect smaller changes in clearance with a smaller level of imbalance exciting the system, which would be more characteristic of, for example, the behaviour exhibited in worn bearings from a real motor set-up.

4. CONCLUSIONS

The work reported in this paper examines the use of the correlation dimension in condition monitoring of systems with clearance in order to determine whether the use of chaos techniques could provide an alternative route in the machine health monitoring of plant or machinery. As the system considered for this analysis has previously been analysed [11], all observables contributing to the dynamics of the system were known in advance; therefore, calculations could be performed in the full-phase space of the system. In order to accomplish this, the system was considered for three different magnitudes of clearance at two distinct shaft speeds. On constructing the phase space for each case and computing the correlation dimension from these phase spaces, it was discovered that as the clearance was increased, there was a discernible decrease in the value of the correlation dimension for the system. This trend was observed at both the shaft speeds analysed. Hence, it may be possible to determine changing events such as gap clearances through monitoring the correlation dimension of the system.

Practically, however, the method of delays [4] has to be employed to reconstruct most system's attractors. In this case, when embedding space was applied to the same three cases considered for the full-phase-space reconstructions, faithful reconstructions were obtained using the x direction observable alone. However, substandard "attractors" resulted when using purely the y direction observable. It was reasoned that this may have been due to the weakening of the coupling between the two motions as the gap clearance was increased resulting in loss of information about the x direction motion in the y direction motion. In order to improve the quality of the unacceptable reconstructions, the method of delays was modified to incorporate values from an additional observable In this study, it was found that two values from an additional observable were required in order to give "true" embeddings so that accurate estimates could be obtained from the "attractors".

It is evident that it may be feasible to use the correlation dimension in the condition monitoring of systems with clearance. However, reconstructions resulting form the arbitrary applications of embedding space should be analysed with caution as inexact "attractors" could be a possibility with practical systems. Further practical-based analysis, however, would be required to determine whether a trend in the correlation dimension as gap clearance is varied is detectable in more realistic situations. As condition monitoring schemes usually monitor data relating to the outer casings of the components which are of interest, it would have to be established through further analysis whether the techniques described in this paper could be extended and employed in a manner suited to these conditions imposed in realistic monitoring set-ups.

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APPENDIX A: NOMENCLATURE

c_1	primary damping coefficient
c_2	secondary damping coefficient
$C(r)$	correlation function
D_2	correlation dimension
$H(\ )$	Heaviside function
i	summation index
j	summation index
k_1	primary rotor stiffness
k_2	secondary rotor stiffness
m	mass of rotor
N	number of data points
r	radius of the hypersphere
R	radial co-ordinate of the rotor
V_{Tang}	tangent velocity
x	co-ordinate of the rotor
y	co-ordinate of the rotor
δ_1	offset in the x direction
δ_2	offset in the y direction
δ_r	radial clearance
μ	coefficient of friction
v	power-law index
v_1	primary damping ratio

ν_2	secondary damping ratio
ρ	mass ecentricity
τ	time delay
ω_1	primary natural frequency
ω_2	secondary natural frequency
Ω	rotor angular velocity
$(\dot{})$	differentiation with respect to time